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ART. I. — 1. *Physical and Celestial Mechanics.* By BENJAMIN PEIRCE, Perkins Professor of Astronomy and Mathematics in Harvard University, &c. *Developed in Four Systems of Analytic Mechanics, Celestial Mechanics, Potential Physics, and Analytic Morphology.* Vol. I. *A System of Analytic Mechanics.* Boston: Little, Brown, & Co. 1855. 4to. pp. xxxvii., 496.

2. *Theory of the Motion of the Heavenly Bodies moving about the Sun in Conic Sections; a Translation of* GAUSS'S "*Theoria Motus.*" With an Appendix. By CHARLES HENRY DAVIS, Commander U. S. N., Superintendent of the American Ephemeris and Nautical Almanac. Boston: Little, Brown, & Co. 1857. 4to. pp. xvii., 326, 40.

IN a recent number of this journal we spoke of the work of the great Irish mathematician as a magnificent effort of the imagination. The publication of the most valuable original mathematical treatise as yet written in America, has suggested to us a different train of thought upon the Reason in Mathematics.

Professor Peirce is distinguished in all his writings, from his Elements of Geometry to his Analytic Mechanics, by a peculiarity in his modes of proof. His demonstrations are always concise, and remarkable for the directness with which they attain their end. In the present volume is an instance in which a proposition is established by a few lines of argument, as clearly and incontrovertibly as by the dozen pages which the first propounder of the theorem employed in its proof. Such facts suggest an inquiry into the nature of the difference between the reasoning of different writers, and into the effect which the study of different mathematical writers may have upon the habits of thought in the student. There is a well-known anecdote of Napoleon's disappointment, when he took Laplace into his cabinet, and found that this distintinguished mathematician was a wholly unsafe guide on questions of state policy, seeking to introduce the spirit of the infinitely small into the government. This anecdote is frequently used to cast discredit upon the mathematics, as a means of mental discipline. There are, indeed, upon this, as upon most other subjects, two entirely opposite views current among those who have considered the question. Some attribute to mathematical studies an almost omnipotent power in strengthening the judgment and giving clearness and vigor to the logical faculties; others are inclined to ascribe to these pursuits the invariable effect of rendering a reasoner a slave to the mere forms of logic, and making him forget the superior value of the dictates of common sense.

In order to decide intelligently upon the merits of this question, let us first analyze the process of reasoning. In what does it consist? What do we mean by saying that we have proved a truth? The elements of logic deal with three distinct things, — the conception of ideas, the perception of a relation between ideas, and the perception of a relation between those relations; that is, with terms, propositions, and inferences. Or, to use grammatical language, logic deals with subjects and predicates, with propositions, and with the mutual connection of propositions. Different minds are capable of comprehending these three classes of things with different degrees of clearness. The power of grasping an idea is essentially different from the power of perceiving a truth; and the power of drawing an inference is essentially different from either of the other two. Now the part of reasoning which is common to all departments of human thought is simply this third part, drawing inferences. The ideas or things concerning which we reason are different in each different science; the truths, that is, the relations between these things or ideas, must also differ in each department of inquiry; and the relation between the relations, being the only abstract thing, is the only one which is the same in all sciences and all subjects of thought. But although the process of reasoning is the same, upon whatever subject we reason, it will not appear so unless we are capable of perceiving all kinds of truth, and grasping all sorts of ideas, with equal facility. The same argument which, in a common matter, seems perfectly convincing, may on an unfamiliar subject seem wholly irrelevant. An example from the Algebra of our author will explain our meaning without obliging us to introduce technical mathematics. There is a theorem of Arbogast which Peirce has demonstrated in the space of a single duodecimo page. The students of Harvard University are seldom able to master this demonstration. The whole difficulty of it, however, lies in about half a dozen lines. Year after year, class after class of the most intellectual young men of New England admit themselves to be unable to comprehend the reasoning of this short paragraph; and yet its whole reasoning, if we substitute the names of places for the names of mathematical quantities, amounts to this: — If we can go from Boston to New York, and if we can go from Portland to the City of Notions, we can go from Portland to New York, since the City of Notions is only another name for Boston. No reader will perceive that there is any difficulty in this argument, as it now stands; and in the form in which Peirce gives it, the difficulty is not really, but only apparently greater. From P we can obtain P, and from Q we can obtain Q; but P equals Q, and therefore from P we can obtain Q. No reader, except one who has had an algebraic training, will feel that this argument upon the letters is as clear as the one upon the cities, and yet the argument is precisely the same. The only difficulty is in conceiving, first, what P and Q may stand for, and consequently what can be the relation between them and P and Q.

The first great logician, Aristotle, reduced all reasoning to a syllogistic form, and all syllogisms are equivalent in their nature to this inference, — that what is true of all members of a class, is true of each member of it. Thus the most direct form of the syllogism would be: — A certain thing is true of a certain class of objects; this object belongs to that class; and therefore that thing is true of it. The whole doctrine of syllogisms has been beautifully illustrated by three geometrical figures. Every assertion or denial may be symbolized, for example, by the assertion or denial that a certain square is included in a circle, the universal propositions putting the square wholly within, or wholly without, the circle, and the particular affirmation or denial leaving a part within, and implying that a part is without. All reasoning may then be reduced into syllogisms such as the following: — The whole of the square is within the circle, the whole of the triangle is within the square, and therefore the whole of the triangle is within the circle; The square is not within the circle, part of the triangle is within the square, and therefore part of the triangle is not within the circle.

Later writers upon logic have regarded the syllogism as simply a test of an argument, and have very clearly shown that it is not the form in which arguments are consciously put in the process of thinking, but only the form in which sound arguments may always be put, for the purpose of testing their soundness. Some have particularly endeavored to show that inductive reasoning is not readily put into a syllogistic form, and have therefore sought to establish a new logic for the examination of inductive reasoning. We have, however, for many years, been in the habit of presenting the whole matter of logic in what we consider a more simple and popular form than the syllogistic, and at the same time no less sound. It has moreover the advantage of embracing equally those higher forms of reasoning used in modern science, both inductive and demonstrative, which are with so much difficulty brought into a series of syllogisms. Instead of comparing reasoning to the placing of three figures within or without one another, we compare it to the laying of a path from points that are known to points that are unknown. The known points are self-evident or admitted propositions. The successive steps of the path are self-evident or admitted relations. The unknown points to which the path conducts are the propositions to be proved. In geometrical synthetic demonstration we pass by a series of self-evident steps from the simplest self-evident truths to the highest deductions of the science; while in geometrical analysis we pass, through self-evident steps from the proposition to be proved, down to the simplest axioms. In either case we show an actual dependence of the complex proposition upon the axiom. Now it is true that each step of this process may be thrown into the syllogistic form, but it is equally true that no man is, in reasoning, conscious of this reduction; — the actual process of the mind is, to pass directly from proposition to proposition, without justifying the step by that generalization, the result of which is called in the syllogism the suppressed premise. In every process of reasoning there must be at some point a resting upon first truths. Sound reasoning cannot run in a perpetual circle, neither can it be an infinite series of answers to the question, Why? To such a series there must at length come the answer which we are so often forced to give to children, — that it is, because it is. There can be no disputing concerning the primary conceptions of the mind, or the perception of the primary relation between those conceptions. There are men incapable of grasping the geometrical conceptions of a point, a line, and an angle. There are others who seem equally unable to grasp the ideas of truth, justice, and duty. In neither case does the inability of a few to form such ideas make them any less clear and definite to the majority of men. But some of those who can conceive of points, lines, and angles do not acknowledge the fact of their existence in space; they allow to them only an ideal existence. Those who believe them to be really entities cannot hold any argument upon the point with those who deny their existence; because they esteem their existence to be a self-evident truth. Nor can one who believes in the reality of the difference between right and wrong, enter into any debate with those who deny that difference. In Jouffroy's Introduction to Ethics, while he reviews and rejects those systems of philosophy that destroy the reality of ethical distinctions, he does not enter into an argument to prove their reality; for the existence of a distinction between right and wrong is to his mind a self-evident truth. In the matter of simple conceptions, there can be only the difference between grasping and not grasping the idea; and in the matter of self-evident truths, only the difference between seeing and not seeing them. He who sees may be right, or he who fancies he sees may be wrong; but in either case the matter is beyond argument, and can be reached only by patient meditation or direct observation. In like manner, in self-evident connections or trains of reasoning, there must be steps so simple as to admit of no further debate. We see their truth or their fallacy in the particular case before us, and the truth or fallacy is not usually made more apparent by throwing the proposition into general terms, and thus converting it into an expressed premise. The syllogism is of value to detect a fallacy only when we are reasoning about subjects with which we are not perfectly familiar, and in which we therefore carry the cloudiness of our conception of the truths themselves into our perception of their relations; as, in the case quoted from Peirce's Algebra, our cloudiness of conception of the meaning of P and Q impairs our ability to understand the perfectly clear reasoning employed upon them. Now these steps, so simple as to admit of no debate, are always capable of being verified by being resolved into the inference of a conclusion lawfully drawn from the premises of a syllogism; but it is by no means necessary thus to resolve them in order to feel their justness, neither is that resolution strictly a psychological analysis of the operation of the thought.

The representation of reasoning as the process of connecting the truth to be proved, by a series of self-evident steps, with self-evident propositions, has the advantage of including inductive reasoning in natural science, and demonstrative reasoning in mathematics, as well as reasoning upon ordinary subjects. It has the advantage of embracing a connected series of syllogisms in one simple definition. But its principal advantage is, as we have said, that it is more closely conformed to the actual operation of the mind in reasoning. Take, for example, the proposition that, if a square is surrounded by a rope, and there is also a rope crossing the square upon each diagonal, there must be at least two pieces of rope. We first form a clear conception of a square, surrounded by a single line, and crossed by two diagonals, each of a single line; and a self-evident connection of ideas shows us that three lines will proceed from each of the four corners of the square, that is, a line to each of the other three corners. Artifice may put this into a syllogistic form, to prove that three lines will proceed from each corner, but it will surely not be a natural process. Again, a self-evident connection of ideas shows us that there must be an end of the line at each corner of the square; nor do we here naturally say, Wherever an odd number of lines radiates from a point, one of the lines must end there. This general proposition may be framed, but in framing it we should use our perception of its truth in individual cases; the perception of its truth in this case must therefore be independent of, and antecedent to, our perception of the truth of the general proposition. Again, a self-evident connection of ideas shows that there must be four ends to the string about this square under consideration; nor do we here use the major premise that every square has four corners; our thought is confined to the particular square which is here surrounded by a string and crossed by two diagonal lines. Finally, a necessary connection of ideas shows us that these four ends must belong to two pieces of string; and that, also, without the mind's necessarily asserting that every piece of string has two, and only two ends. This axiom has probably very seldom been framed as a justification of any reasoning about ropes.

If, then, the dependence of one proposition upon another is not usually conceived, in the actual process of reasoning, as the including of the members of the class under the class, in what form is it conceived? We answer, that it is the perception that *those members* have the property of the class, without perceiving the fact of their being included in the class. In saying that C must be A, because it is B, we usually only see that the B which is in C is A, and do not necessarily see that all B is A; which would be the major premise if we attempted to put our argument into syllogistic form. The dependence of one proposition on another is a virtual relation of an included part to its whole, but this is not the only form which it actually wears to the eye of the reasoner. The fact that the Eastern and the Boston and Maine railroads cross in Somerville, and meet in South Berwick, shows that they cannot both run in straight lines; but he who makes this inference, and sees its truthfulness, does not frame any general proposition by which he justifies his inference. He simply sees that, if both roads continued straight after their crossing, they would run half round the world before they met again. The idea of a straight line being a great circle of the sphere, and great circles cutting each other only at the opposite ends of a diameter, does not enter his mind. He perceives its truth in this particular case; but as it is an abstract truth, it cannot be expressed in the particular and special form in which he sees it, but must in its expression include all cases of a similar relationship of ideas. The natural process of thought is simply to conceive of the two railways as diverging; and it is only in the attempt to put the thought into language that we say, two straight lines can cross each other but once.

The acknowledgment that Aristotle's *dictum de omni et nullo* is the basis of every inference, shows, of course, that we conceive only one relation between propositions to be available in reasoning; that is, the relation of a part to a whole. One proposition can depend inferentially upon another only if it is virtually included in that other. The doctrine of syllogisms, therefore, necessarily includes the whole doctrine of logic, in its proper sense. And, conversely, if the subject of logic be fully and fairly discussed, it must present itself in the syllogistic form. All that we claim for our mode of presenting reasoning, under the analogy of a road from the premise to the conclusion, is, that it is a more popular and natural form, which includes enthymemes and sorites in the same definition with the regularly expressed syllogism. The value of this claim is apparent, when we perceive that most reasoning is enthymematical as it originally presents itself to the mind of the reasoner, whatever form it may assume in utterance. No general truth or proposition will be admitted, unless particular cases under it have first been admitted, and it is the perception of the particular case, and not the perception of the general truth, that gives the real efficacy to an enthymeme. To illustrate again by the simplest possible case (in *barbara*), we recently saw a swarm of acari upon a window-pane, and, in reasoning upon their origin, assumed at once that they required food of some kind. The enthymeme that passed through the mind was, These are animals, and therefore require food. But the connection between these two propositions was evident to the mind, from the difficulty of conceiving of those special organisms before us, with the fluids of their bodies thus exposed to evaporation, as kept in life without a supply of food. The argument had its strength from a perception, in that individual case, of the truth that the animal life requires food. It could be thrown into a syllogism by asserting, as a major premise, that every animal requires food; but that premise would be granted only from a consideration of the fact that some particular animals need food, and that this arises from properties which they have in common with all other animals.

Now the peculiarity of Peirce's mathematical reasoning, which gives his demonstrations such exceeding brevity, seems to us to consist in his resort to this direct and natural mode of thought, and his freedom from the restraint of conventional forms of demonstration. We do not mean simply that his reasoning is, in logical language, enthymematical, suppressing one premise; this is common to all reasoning. But we refer to the habit of using simple conceptions, axioms, and forms of expression, without reference to established usage. Into his Geometry he introduces the idea of direction to simplify the doctrine of parallel lines and angles, and the idea of the infinitesimal to simplify his treatment of the circle. In his Algebra he shortens the demonstrations of many theorems by introducing from the Calculus the idea of variables and constants, and the rate of change in functions. In his Trigonometry he frees his calculations from all dependence upon the radius by defining his trigonometrical functions as simply arithmetical quotients of lines. In his Calculus he preceded Cauchy in his improved notation, freeing the differential coefficient from the expression of the differential denominator.

Thus also, in the present work, he appeals directly to self-evident truths, whether they are among the acknowledged axioms of geometry or not; assuming, for instance, that "a line which neither passes out of a finite space, nor ends within it, must be a re-entering curve." While ancient geometers and their disciples appear to have a double object in view, the discovery of truth, and its reduction to the time-honored forms of demonstration, Peirce, as the representative of the modern school of mathematicians, seeks to discover and to present truth, in whatever form seems clearest and most beautiful to him.

The number of self-evident truths in geometry, and in other sciences, is doubtless usually underrated. That is self-evident which can be seen by direct observation, either of the material phenomenon, or of the mental conception. Most persons are content without an analysis of the foundations on which their acceptance of a truth stands; and most of those who attempt to analyze, are content to be guided by the analysis of others. It is only a few great spirits who, by actual analysis of ideas, enlarge the number of truths admitted to be self-evident. The proposition that the sphere is the maximum among solids of equivalent surface, requires, for its reduction to ordinary geometrical axioms, about two hundred distinct steps of inference; that is, the proof of its truth could be put into syllogistic form, employing the ordinary geometrical axioms, only by the use of about two hundred syllogisms. But every boy who has blown a soap-bubble, can prove the same truth by three or four steps from self-evident truths. The air enclosed in the expanding bubble, endeavoring to make most room for itself within the surface enclosed by the film of soap-water, will evidently succeed best when allowed unimpeded expansion in every direction; but it will then expand equally far in all directions from its centre of bulk, and thus produce a sphere. No sound mind can object to the real validity of this reasoning; it is a tacit assumption of Leibnitz's principle of the sufficient reason. This great man showed his greatness in his desire to enlarge the number of admitted self-evident truths; and, however desirable on other grounds it may be to give a geometrical demonstration of problems of isoperimetry, this mechanical demonstration, from Leibnitz's axiom, is sufficient for the satisfaction of a seeker after truth.

Maupertuis's principle of the least action, which Peirce honors as a grand theological idea, is another means of greatly shortening the process of demonstration, and of anticipating the results of observation. "It is inconsistent," says Maupertuis, "with our idea of divine wisdom, to suppose that God would use more power than was necessary to accomplish a given end." Hence we may assume, that, in every operation of nature, the law is such as to require a minimum amount of force. Professor Snell, of Amherst College, ingeniously develops all the ordinary laws of optics, with regard to the reflection and refraction of light, from this single principle. Thus, between two points in the same medium, the shortest path will require the least power, and therefore, under such circumstances, light will move in straight lines. But if, in going from one point to another, the light is required to touch also a given surface, the path to the surface and then to the other point must be the shortest possible; which geometrically gives us the observed law of reflection. If the points are in different mediums, the path, instead of being straight from point to point, must be, in general, lengthened in the rare medium, and shortened in the dense one, in such proportion as to make the whole exercise of force the least possible; and this will lead, by geometrical deduction, to the same laws of refraction that have been discovered by observation.

The effect of mathematical studies upon the student will evidently depend partly upon the student's own habit of thought, and partly upon the style of the geometer to whose writings he gives most attention. In the first ten duodecimo pages of our author's Integral Calculus is contained, in a clear and definite form, every principle contained in the first fifty pages of La Croix's quarto. To study these briefer and equally conclusive modes of presentation must surely contribute to energy and vivacity of mind, and to directness and force of mental action, more effectively than plodding over the prolix and tedious dilatation of the same themes. To be refreshed with the continual appeal to a variety of self-evident truths, must enlarge the sphere of thought, and give freedom of play to the intellect, more surely than the rigid attempt to make all mathematical truth depend upon eight or ten axioms. But the lover of mathematics must acknowledge that the study of his favorite science has not any direct tendency to render our conceptions of moral and political subjects more definite and vivid, or our perception of the relations between them more accurate. Its direct effect upon the powers of conception and perception is confined to the physical sciences, in which the laws of space and time hold sway. Here the value of the mathematics is much greater than is commonly supposed. They are usually pursued as a discipline of the reasoning powers; but, as we have shown in a previous article, they cultivate the imagination also, to a high degree. The study of geometry, developing the power of conceiving of forms, and the study of algebra, developing the power of conceiving of relations of quantity, will assist the student in all possible physical inquiries. But the relations of men to one another, and to the spiritual ends of the universe, are not directly connected with space and time; and the understanding of these matters is not directly assisted by mathematical pursuits. In regard to reasoning, however, the case is different. Relations between propositions, being purely abstract, (since propositions themselves are but the relations between ideas,) are always general and comprehensive in their form, and the habit of tracing the dependence of propositions upon each other, that is, reasoning, in the usual sense of the word, upon whatever subject, strengthens the power of reasoning upon all other subjects. All reasoning can be reduced to a syllogistic form, and the laws of the syllogism are entirely independent of the subject-matter of the argument. Nevertheless, the student is not to forget that practical reasoning is something else than a series of syllogistic forms. Practically, only one premise is usually present to the consciousness of the reasoner. The rapidity and assurance with which we could form these suppressed premises, if we chose so to do, varies greatly in different trains of argument; and it is this variation that constitutes the chief distinction between demonstrative and inductive reasoning. In demonstration, the examples that can be classed under the major premise are exceedingly simple, and the mind can run through them with great rapidity. Thus, when we say that two straight lines cannot enclose a space, the mind conceives with exceeding rapidity of two straight lines crossing each other at an infinite variety of angles, perceives that they never cross each other in a second place, and instantly leaps to the conclusion that they cannot so cross each other. What is seen to be true of a myriad of cases surveyed instantaneously by a mental glance, is confidently assumed to be true of all possible cases. But in inductive reasoning, the mind proceeds more slowly in the investigation of examples, and infers more cautiously the general truth, because from the slowness of its operations it must base its conclusion upon a smaller number of facts. Thus, Newton's binomial theorem may either be demonstrated by an appeal to self-evident truths of the differential calculus, or gathered by induction from the example of a binomial raised to two or three different powers. Now it is a grave mistake in reasoning, if we attempt to apply the demonstrative type to subjects in which it is inapplicable, and scarcely a less grave error to be content with induction where we can gain demonstration. It is true that the boundaries of these divisions of the reasoning process are not, and cannot be, sharply defined; since, if they were, all reasoning would no longer be the same process; — but between the extreme ends, there is a marked difference: it is folly to attempt demonstration as to most practical questions in political economy, and it is folly to be content with induction from experience in matters of moral philosophy. The mathematics supply us with numerous examples of both types of reasoning. While in all the simpler departments demonstration is attainable, in the highest walks we must be content with induction. It is a fine discipline for the student's mind to learn when he may be content with induction, when he may accept its conclusions with confidence, and when they are to be received as simple probabilities.

From the view which we have given of demonstration and induction, it will be seen that we cannot agree with those who hold it to be impossible to demonstrate the existence of a being. The axioms of geometry and of other sciences seem to us to differ from the truths of observation only as to the mode in which the truth is perceived. A sphere is, to us, as real an entity as a globe; and, on the other hand, it is as absolutely certain that globes are seen, as that spheres are conceived of. An argument based upon the perception of globes, it seems to us, therefore, may be as strictly demonstrative, as one based on the conception of spheres. If the distinction between self-evident truths and truths of observation is founded upon the assumption that the contradiction of the former is absurdity, and of the latter, error, we reply, that absurdity is simply the maximum of error. To limit the office of demonstration, with Dugald Stewart, to the work of showing that an hypothesis is consistent in all its parts, would be to render it futile. In its true office it deals with ideas of perception as well as of conception; and the certainty with which we infer the existence of a fellow-being, or of a Supreme Mind, is as assured and as reasonable as that with which we infer the truth of the Pythagorean proposition, whether it be technically called demonstration or not.

A careful study of the mathematics will also show the student that demonstration is not always conclusive, but needs to have its validity demonstrated. In other words, the human mind is not capable of attaining certainty, even upon mathematical points, at a single step; but the most careful mathematicians are liable to error, arising from each of the three possible sources; from inaccuracy of conception, from error in the supposed perception of a self-evident truth, and from mistake in drawing a supposed self-evident inference. So far, therefore, from making man proud of his own powers, the mathematics teach the wisest student humility and caution in the investigation of every truth. The fable of the knights, who disputed whether the shield was of gold or of silver, has been more than once realized in the history of the most certain of sciences; and a warm controversy as to the value of a quantity has been finally ended by showing that it could assume either of the disputed values. The most illustrious example, in late years, is that of the planet Neptune, which

Le Verrier demonstrated to be at thirty-seven times the earth's distance from the sun, but which was actually found to be at only thirty times that distance. The analytical power of Peirce explained this difference, by showing that a planet in either place would produce those perturbations of Uranus upon which Le Verrier based his demonstrations.

Mathematical reasoning is frequently illustrative of reasoning upon entirely different subjects, and although Locke tells us that it is a weakness of the human understanding which leads us to attempt to fortify its convictions by a comparison with analogous cases, yet it is a weakness inherent, we suspect, in human nature, and the mathematical student may be pardoned if he indulges in it, while speculating upon moral and political subjects. That great problem, for example, for all finite intellects, to reconcile the conception of man's free agency with that of God's foreknowledge, may have its parallel even in geometry, in which also we can find incontrovertible truths, neither of which can be doubted, and which are yet diametrically opposed to each other. That two parallel lines, a mile apart, may have a third line drawn between them by such a law that it shall continually approach one of those parallel lines, and yet end in the other, is as palpable a contradiction of ideas, as can be presented to the mind; and yet it may be easily demonstrated that such a line can be drawn, being in fact a hyperbolic spiral.

A striking lesson may also be derived from the fact, that the most perfect solution of a problem is not always the most useful. In the various theorems for discovering real roots, those that are the most imperfect are, in many cases, practically of the most value. As we approach the higher branches of mathesis, similar, but stronger, instances grow frequent. In Professor Peirce's volume now before us, the most general and exhaustive solutions are often entirely unfitted for practical applications; so that in numerical examples approximative methods alone can be employed. Indeed, in this sense, all that is practical must be approximative. For every law of quantity is in general continuous; while every practical application of that law, depending as it does upon arithmetic, must be discontinuous and imperfect. The man to whom these facts are familiar, cannot but feel that they illustrate all social and political problems; and he is thus led to labor with more patience and long-enduring hope for the improvement of practical means for applying to human society the perfect laws of Christian ethics. The same subject, viewed from a slightly different stand-point, may illustrate the difference between the poetical and the practical man. One mathematician, with impulsive genius, finds perfect solutions of every problem, which may or may not be applicable to questions of numerical computation. Another, with more patience and industry, but with less genius, either reduces those solutions to a practical form, or else, assailing the subject by his own method, finds some awkward and inelegant, but available solution. Both these men are useful, and to be held in high honor in their respective departments of mathematical labor. The like difference is to be found in minds, occupied in other departments of human thought. A poet utters the widest views of statesmanship, which may or may not be in a form directly applicable to action. A statesman, having less genius, may either put the ideas of the poet into a practical form, or present his own views upon the subject, in a manner less agreeable and yet directly available for practical use.

The error which is usually committed in regard to mathematical studies lies in the mode of their presentation. While other branches of learning are presented in attractive guise, with notes, and anecdotes, and illustrations, the mathematical text-book is usually one of dry formulas alone. The only exception to this, in American school-books, besides those on arithmetic, and one or two partial exceptions in geometry, is in a book on Analytical Geometry by Mr. J. M. Peirce, a son of Professor Peirce. In that treatise, the subject is enlivened by historical allusions, metaphysical comments, and speculations on the infinite, which, without materially enlarging the bulk of the volume, or interfering with its main purpose, render it attractive to the student. Now could a student, qualified by nature for the pursuit, take up in succession all the principal branches of modern mathematics with the aid of similar treatises, catching the spirit as well as the form of the various processes, it would hardly fail to enlarge his breadth of view on every subject. So far from the mathematics leading a man to introduce the infinitely small into all departments of reasoning, a more extensive and just view of mathematical reasoning shows, more forcibly than any other one science can show it, that each subject requires its own peculiar mode of treatment. A proposition in simple arithmetic or geometry may be proved in a variety of ways, of which some are undoubtedly better than others. As we ascend higher in the general science of space and time, the variety of ways in which a proposition may be established increases, and the difference of value in the various modes increases. The difference between a few lines and a dozen pages of argument to establish the same proposition, is certainly enormous. But the difference is sometimes vastly greater even than this. Propositions are sometimes absolutely incapable of proof in a purely geometrical mode, which may nevertheless be readily demonstrated by algebra; others require that algebra should take the form of analytical geometry; others demand a trigonometrical solution; and so on, through all the multiform *calculuses* of the nineteenth century. No imagination can invent a new engine of mathematical investigation which shall not in some particular problem present peculiar advantages for extorting truth. And scarcely any imagination is so dull that it cannot propose an original problem, apparently simple, and yet impregnable against all the engines that have yet been invented. But can the man who has thus seen that the same reasoning is not applicable to every subject, even in the limited sphere of mathematics, be so likely as before to look at all things from one point of view? Will he not, if at all tractable, learn that a special problem requires a special treatment? How astonishing is the conceit of those who presume to dogmatize freely upon matters to which they have given no particular study or thought! The intricate and thrice-complicated questions of currency, trade, credit, and the effect of legislation upon them, are authoritatively settled by politicians, after five minutes' attention to the subject, on the strength of an apparent demonstration of one or two steps from self-evident truths; while political economists scarce venture an opinion after years of patient investigation.

The Preface to the volume before us will present a more complete view of Professor Peirce's object in writing the book than can be given in other words. He says: —

> "I have re-examined the memoirs of the great geometers, and have striven to consolidate their latest researches and their most exalted forms of thought into a consistent and uniform treatise. If I have, hereby, succeeded in opening to the students of my country a readier access to these choice jewels of intellect, if their brilliancy is not impaired in this attempt to reset them, if in their new constellation they illustrate each other and concentrate stronger light upon the names of their discoverers, and still more, if any gem which I may have presumed to add, is not wholly lustreless in the collection, I shall feel that my work has not been in vain. The treatise is not, however, designed to be a mere compilation. The attempt has been made to carry back the fundamental principles of the science to a more profound and central origin; and thence to shorten the path to the most fruitful forms of research. It has, moreover, been my chief object to develop the special forms of analysis, which are usually neglected, because they are only applicable to particular problems, and to restore them to their true place in the front ranks of scientific progress. The methods which, on account of their apparent generality, have usually attracted the almost exclusive attention of the student, are, on the contrary, re-established in their true position as higher forms of speciality."

The first of these endeavors has been eminently successful, and is particularly useful. The lapse of two centuries, since mechanics became a science, had rendered many of the most valuable results of the early Pelasgi of the infinitesimal methods so antiquated in their dress, that they were repulsive to the modern student, and partly unintelligible. All is here brought forward and stated in the latest forms of the science, so as to be at once available for the young student's use.

The gems of which our author speaks so modestly will be more highly praised by others. A method of partial multipliers, which includes Euler's, Lagrange's modifications of Euler's, and Jacobi's principle of the last multiplier, as but special cases; a complete discussion of the pendulum, both from an *a priori* view and from experiment; a full and general solution of the problem of central forces; a more complete analysis of rotation, including the problems of the top, the gyroscope, the hoop, &c., than has ever before been given; an examination of the catenary, embracing many new forms; a general solution of the question of perturbations and vibrations; a view of the motion of a body free to move in a rotating curve, — these are a few of the brilliants which seemed to us, in a rapid glance over the collection, to be both new and lustrous. We are also inclined, with our author, to attach a very high value to his attempt to restore special methods to a prominent place among the means of scientific progress. It is in accordance with the views which we have here given of reasoning, as based upon special, rather than general truths; it agrees with the spirit of modern philosophy, as illustrated by the earlier laborers in the revival of science, and proclaimed by that Balaam of the scientific Exodus, who said that the true method was to ascend from particulars to universals, before we descended from universals to particulars; it accords with the whole spirit and intent of the intellectual discipline of this world, in which our attention is first caught by glittering specialties, and does not perceive wide and universal truths until mature age. The best general solutions are those which have been obtained from the examination of particular examples. Nay, even the best general methods of solution have frequently been discovered from a special attack upon some individual case of difficulty. No men are rendering more veritable service to truth, to science, and to their fellow-men, than those who are making patient and thorough investigation of some simple natural phenomenon, or some single scientific problem of a limited character. From such labors come the great discoveries and great inventions of the nineteenth century; from such came the equally wonderful growth of the eighteenth and the seventeenth.

We have quoted a large part of the Preface, and will close our remarks upon this work with the closing words of the last chapter, in which the author announces the subject of the succeeding volumes, Celestial and Physical Mechanics.

"In the present volume the attempt has been made to concentrate the more important and abstruser speculations of analytic mechanics clothed in the most recent forms of analysis, and to make a few additions, which may not be rejected as unworthy of their position. Much, undoubtedly, remains imperfect and unfinished, for it cannot be otherwise in a science which is susceptible of infinite improvement; and much must soon become antiquated and obsolete as the science advances, and especially when we shall have received the full benefit of the remarkable machinery of HAMILTON'S *Quaternions.* But it is time to return to nature, and learn from her actual solutions the recondite analysis of the more obscure problems of celestial and physical mechanics. In these researches there is one lesson, which cannot escape the profound observer. Every portion of the material universe is pervaded by the same laws of mechanical action, which are incorporated into the very constitution of the human mind. The solution of the problem of this universal presence of such a spiritual element is obvious and necessary. THERE IS ONE GOD, AND SCIENCE IS THE KNOWLEDGE OF HIM."

It is a remarkable proof of the real and rapid progress of Astronomy in our country, that the same publishing house should have issued almost simultaneously two costly works upon that science;* for a treatise on Analytical Mechanics is, in one sense, but a preparation for one on Celestial Mechanics. The translation of Gauss's Theoria Motus, by Captain Davis, is a very welcome addition to astronomical literature. The original volume was nearly out of print; and the language in which it was written is now but little used in scientific writings. But Gauss had so well solved the problem of determining the orbit of a heavenly body from three observed places, and of determining the orbit which would best satisfy any number of observations, that the lapse of half a century had only made it more certain that his work would ever remain the standard and classical treatise on the subject. Captain Davis has, therefore, to a faithful and elegant translation, simply added as an appendix whatever could be gathered from the labors of astronomers gleaning in this field so thoroughly harvested in the body of the work. Among these additions will be found not only contributions from European astronomers, such as Encke, Bessel, Gauss himself, Le Verrier, &c., but also important articles from Professor Peirce, and useful tables from Professor Hubbard. The present edition leaves, indeed, nothing to be desired, everything of recognized value, in the computations of planetary and cometary orbits, being comprised within its covers. The expense of publication is borne partly by the Nautical Almanac and partly by the Smithsonian Institution.

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* It should be emphatically stated that the publication of Professor Peirce's volume is due to the disinterested munificence of Messrs. Little, Brown, & Co., the subscription having failed by six or seven hundred dollars of meeting the cost of publication, — a deficit which the sales of a quarter of a century would hardly replace.

ART. II. — 1. *The Life of George Stephenson, Railway Engineer.* By SAMUEL SMILES. Second Edition. London: John Murray. 1857. 8vo. pp. 528.
2. *The Same.* From the Fourth London Edition. Boston: Ticknor and Fields. 1858. 12mo. pp. 486.

MR. SMILES'S Memoir is worthily written, in a clear and flowing style, apt in illustrations of character, and presenting its subject as a truly great man, who won fame and fortune solely by untiring industry and dauntless integrity.

George Stephenson was born on the 9th of June, 1781, in the colliery village of Wylam, about eight miles west of Newcastle-on-Tyne. His parents, Robert and Mabel Stephenson, were an industrious, hard-working couple, who lived in a two-storied red-tiled building, portioned off into four laborers' dwellings, with unplastered walls, clay floors, and bare rafters exposed overhead. Their bread depended entirely on their daily wages, and, as an aged neighbor said of them to Mr. Smiles, "they had very little to come and go upon; they were honest folk, but sore haudden doon in the world." A tradition indeed existed that Robert Stephenson's father and mother came over the Scottish border after the loss of considerable property; but however that might be, their son remained always a humble workman to the end of his days. He was fireman of the pumping-engine at the Wylam Colliery, and thus it happened that George Stephenson's earliest recollections were of a steam-boiler. His father was an exceedingly amiable man, remembered in his own circle for his curious love of romance as